Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
For consideration: not mandatory.page 5 line 41  1700’s -->  1700s. (It’s a plural, not a possessive. However, there is also a convention among some writers that plurals of numbers or symbols are given an apostrophe: 5’s, x’s.)page 16 line 40-41 types of solutions --> types of solution (the singular is type of solution, and the plural goes with the main word ‘type’. Compare ‘powers of attorney’.) The authors’ version is common in North America, however, so this might be deemed an acceptable variant.

Review form: Reviewer 2 (Steven H. Strogatz)

Is the manuscript scientifically sound in its present form?
Yes

Are the interpretations and conclusions justified by the results?
Yes

Is the language acceptable?
Yes

Is it clear how to access all supporting data?
Not Applicable

Do you have any ethical concerns with this paper?
No

Have you any concerns about statistical analyses in this paper?
No

Recommendation?
Accept with minor revision (please list in comments)

Comments to the Author(s)
This paper is nicely written, carefully explained, and theoretically satisfying. The authors give a unified treatment of models with the same symmetry as Huygens’s famous observations on the synchronization of two pendulum clocks that are coupled through the vibrations of a heavy beam from which they are both supported. It has been a long-standing question to understand why Huygens saw only anti-phase synchronization, but more recent experiments have seen in-phase synchronization, or sometimes the coexistence of both in-phase and anti-phase synchronization. Using the techniques of equivariant bifurcation theory, normal form analysis, and dynamical systems for systems with symmetry, the authors are able to rationalize essentially all the results observed in previous studies. This level of agreement comes as a bit of a surprise, given that the authors’ model for the escapement is qualitatively different from what Huygens used and what several recent authors used in their experiments.

One suggestion is that it would be helpful to show a sketch of the various escapement mechanisms discussed in the text, with words like "pallet" and "scape wheel" and "teeth" all labeled. Otherwise, it will not be easy for readers unacquainted with these terms to visualize
them without a diagram, and no reader wants to have to search for such a diagram elsewhere (e.g., in the book by Landes).

Typos:

p.2 cause cause

p.4 more symmetry THAN (not `more symmetry THAT`)

- Steven Strogatz, Cornell University

Decision letter (RSOS-170777)

25-Jul-2017

Dear Dr Willms

On behalf of the Editors, I am pleased to inform you that your Manuscript RSOS-170777 entitled "Huygens' Clocks Revisited" has been accepted for publication in Royal Society Open Science subject to minor revision in accordance with the referee suggestions. Please find the referees' comments at the end of this email.

The reviewers and handling editors have recommended publication, but also suggest some minor revisions to your manuscript. Therefore, I invite you to respond to the comments and revise your manuscript.

• Ethics statement
If your study uses humans or animals please include details of the ethical approval received, including the name of the committee that granted approval. For human studies please also detail whether informed consent was obtained. For field studies on animals please include details of all permissions, licences and/or approvals granted to carry out the fieldwork.

• Data accessibility
It is a condition of publication that all supporting data are made available either as supplementary information or preferably in a suitable permanent repository. The data accessibility section should state where the article's supporting data can be accessed. This section should also include details, where possible of where to access other relevant research materials such as statistical tools, protocols, software etc can be accessed. If the data has been deposited in an external repository this section should list the database, accession number and link to the DOI for all data from the article that has been made publicly available. Data sets that have been deposited in an external repository and have a DOI should also be appropriately cited in the manuscript and included in the reference list.

If you wish to submit your supporting data or code to Dryad (http://datadryad.org/), or modify your current submission to dryad, please use the following link: http://datadryad.org/submit?journalID=RSOS&manu=RSOS-170777

• Competing interests
Please declare any financial or non-financial competing interests, or state that you have no competing interests.
• Authors’ contributions
All submissions, other than those with a single author, must include an Authors’ Contributions section which individually lists the specific contribution of each author. The list of Authors should meet all of the following criteria; 1) substantial contributions to conception and design, or acquisition of data, or analysis and interpretation of data; 2) drafting the article or revising it critically for important intellectual content; and 3) final approval of the version to be published.

All contributors who do not meet all of these criteria should be included in the acknowledgements.

We suggest the following format:
AB carried out the molecular lab work, participated in data analysis, carried out sequence alignments, participated in the design of the study and drafted the manuscript; CD carried out the statistical analyses; EF collected field data; GH conceived of the study, designed the study, coordinated the study and helped draft the manuscript. All authors gave final approval for publication.

• Acknowledgements
Please acknowledge anyone who contributed to the study but did not meet the authorship criteria.

• Funding statement
Please list the source of funding for each author.

Please note that we cannot publish your manuscript without these end statements included. We have included a screenshot example of the end statements for reference. If you feel that a given heading is not relevant to your paper, please nevertheless include the heading and explicitly state that it is not relevant to your work.

Because the schedule for publication is very tight, it is a condition of publication that you submit the revised version of your manuscript within 7 days (i.e. by the 03-Aug-2017). If you do not think you will be able to meet this date please let me know immediately.

To revise your manuscript, log into https://mc.manuscriptcentral.com/rsos and enter your Author Centre, where you will find your manuscript title listed under "Manuscripts with Decisions". Under "Actions," click on "Create a Revision." You will be unable to make your revisions on the originally submitted version of the manuscript. Instead, revise your manuscript and upload a new version through your Author Centre.

When submitting your revised manuscript, you will be able to respond to the comments made by the referees and upload a file "Response to Referees" in "Section 6 - File Upload". You can use this to document any changes you make to the original manuscript. In order to expedite the processing of the revised manuscript, please be as specific as possible in your response to the referees.

When uploading your revised files please make sure that you have:

1) A text file of the manuscript (tex, txt, rtf, docx or doc), references, tables (including captions) and figure captions. Do not upload a PDF as your "Main Document".
2) A separate electronic file of each figure (EPS or print-quality PDF preferred (either format should be produced directly from original creation package), or original software format)
3) Included a 100 word media summary of your paper when requested at submission. Please ensure you have entered correct contact details (email, institution and telephone) in your user account
4) Included the raw data to support the claims made in your paper. You can either include your data as electronic supplementary material or upload to a repository and include the relevant doi within your manuscript.

5) All supplementary materials accompanying an accepted article will be treated as in their final form. Note that the Royal Society will neither edit nor typeset supplementary material and it will be hosted as provided. Please ensure that the supplementary material includes the paper details where possible (authors, article title, journal name).

Supplementary files will be published alongside the paper on the journal website and posted on the online figshare repository (https://figshare.com). The heading and legend provided for each supplementary file during the submission process will be used to create the figshare page, so please ensure these are accurate and informative so that your files can be found in searches. Files on figshare will be made available approximately one week before the accompanying article so that the supplementary material can be attributed a unique DOI.

Once again, thank you for submitting your manuscript to Royal Society Open Science and I look forward to receiving your revision. If you have any questions at all, please do not hesitate to get in touch.

Kind regards,
Andrew Dunn
Senior Publishing Editor
Royal Society Open Science
openscience@royalsociety.org

on behalf of Mark Chaplain
Subject Editor, Royal Society Open Science
openscience@royalsociety.org

Associate Editor Comments to Author:
The reviewers and I think this is a great paper, and especially suitable for this journal, but they have a few suggestions for improvement before publication. Could you take a look at the reports and submit a revised version?

Reviewer comments to Author:
Reviewer: 1

Comments to the Author(s)
This paper is nicely written, carefully explained, and theoretically satisfying. The authors give a unified treatment of models with the same symmetry as Huygens's famous observations on the synchronization of two pendulum clocks that are coupled through the vibrations of a heavy beam from which they are both supported. It has been a long-standing question to understand why Huygens saw only anti-phase synchronization, but more recent experiments have seen in-phase synchronization, or sometimes the coexistence of both in-phase and anti-phase synchronization. Using the techniques of equivariant bifurcation theory, normal form analysis, and dynamical systems for systems with symmetry, the authors are able to rationalize essentially all the results observed in previous studies. This level of agreement comes as a bit of a surprise, given that the authors' model for the escapement is qualitatively different from what Huygens used and what several recent authors used in their experiments.

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- Steven Strogatz, Cornell University

Reviewer: 2

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Author's Response to Decision Letter for (RSOS-170777)

See Appendix A.

Decision letter (RSOS-170777.R1)

03-Aug-2017

Dear Dr Willms,

I am pleased to inform you that your manuscript entitled "Huygens' Clocks Revisited" is now accepted for publication in Royal Society Open Science.

You can expect to receive a proof of your article in the near future. Please contact the editorial office (openscience_proofs@royalsociety.org and openscience@royalsociety.org) to let us know if you are likely to be away from e-mail contact. Due to rapid publication and an extremely tight schedule, if comments are not received, your paper may experience a delay in publication.

Royal Society Open Science operates under a continuous publication model (http://bit.ly/cpFAQ). Your article will be published straight into the next open issue and this
will be the final version of the paper. As such, it can be cited immediately by other researchers. As the issue version of your paper will be the only version to be published I would advise you to check your proofs thoroughly as changes cannot be made once the paper is published.

In order to raise the profile of your paper once it is published, we can send through a PDF of your paper to selected colleagues. If you wish to take advantage of this, please reply to this email with the name and email addresses of up to 10 people who you feel would wish to read your article.

On behalf of the Editors of Royal Society Open Science, we look forward to your continued contributions to the Journal.

Best wishes,
Alice Power
Editorial Coordinator
Royal Society Open Science
openscience@royalsociety.org
Dear Editor:

We would like to thank the reviewers for their comments and suggestions. We have modified the manuscript based on these in the following way.

1. All minor typographical errors pointed out by the reviewers have been fixed.

2. A figure (new Figure 2) has been added in Section 2 showing schematics of verge and anchor escapement mechanisms as suggested by the first reviewer. Associated text has also been added in that section.

3. The third reviewer pointed out that one of the hypotheses of the Equivariant Hopf Theorem (EHT) was not satisfied by our system. We thank the reviewer for noticing this. Upon consultation with Martin Golubitsky and Ian Stewart we have determined that although our system does not satisfy the Gamma-simple hypothesis, it does satisfy a condition that is sufficient for the proof of that theorem, with modification, to be valid. We have modified our text in Section 3 to reflect this.

4. The third reviewer also questions why we need the EHT at all. We have added text in Section 3 to explain why we use the EHT: it answers Huygens’ question as to why he saw anti-phase symmetry.

5. The third reviewer points out that we have used a normal form truncated to cubic terms and have not considered the effect of higher order terms. That is correct. We have added text in Section 4 to acknowledge this. However, we expect nearly all of the solutions arising from secondary bifurcations that we discuss are structurally stable and will persist when higher order terms are added, and have indicated so in the text. As for the transition from the 4D to the 6D system, it is based on the Centre Manifold Theorem. The centre manifold is normally hyperbolic by definition, and asymptotically stable in our case. Therefore, trajectories off the centre manifold decay exponentially to solutions on the centre manifold. Therefore, asymptotically, the 6D and 4D systems have the same solutions.

6. The third reviewer requested more clarification regarding what is new in this manuscript compared to our previous paper. We have added text at the end of Section 1 to address this issue. Other than the normal form itself, only the work presented in Section 5 (and not even all of that) is also present in the previous paper. We have also added a sentence in the first paragraph of Section 5 indicating which of that material comes from the previous paper.

Appendix A

2 August 2017
In addition to the above, we have also made changes to Section 5 in the discussion about breather solutions. We have made clearer how the synchronized breather can escape the 2-torus and have added a new figure (new Figure 5) showing a period doubling cascade leading to from a synchronized breather to a chaotic breather.

Finally, we have made a few minor edits for clarity and correctness.

The original synchronized breather video that was uploaded as supplementary material has now been uploaded to Dryad, along with a video of a chaotic breather solution.

Sincerely,

Allan R. Willms

on behalf of co-authors Petko M. Kitanov and William F. Langford.

Allan R. Willms
Dept. of Mathematics and Statistics,
University of Guelph
Guelph, ON N1G 2W1
CANADA